On the Link Excess Life in Mobile Wireless Networks

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Abstract

In this paper, one of the most important parameters in performance evaluation and protocol design of wireless networks, Link Excess Life (LEL), is investigated. An analytical model for LEL is proposed and evaluated. Evaluation of the proposed model leads to the first closed form expression for the excess life PDF. The closed form expression affords ease of further evaluations, e.g., an interesting result is that the average excess life is infinite for the proposed model. The effect of relative velocity distribution on the link excess life PDF is also investigated. The superposition property of LEL is mentioned and used to explore a systematic method for evaluation of the effect of stationary or pausing nodes on link excess life as well as a general method to approximate the link excess life PDF for various relative velocity distributions. Moreover, the effect of the buffer zone on the link excess life PDF in topology control is also investigated analytically. A simulation framework is developed and extensive simulations are performed to validate the results.

1. Introduction

Wireless networks are of the most emerging technologies driven by constant evolution of communication equipments and the proliferation of smaller devices. Thus, performance analysis of such systems in the presence of different factors is an important issue. One of the factors that has a great impact on the design, analysis, and performance of such networks is mobility [1]. In fact, topology of mobile wireless networks is prone to continuous changes which considerably degrades the performance of the routing protocols [2]. In this context, one of the crucial aspects to be addressed is the stability of the established routing paths (also called *path duration*) that is the time interval from when the route is established until one of the links along the route becomes unavailable. Indeed, this metric significantly affects the performance of on-demand routing protocols. When a link is broken down all the paths using this link are torn down too, and new paths must be checked to exist or established which is a resource consuming task [4]. Hence, the performance of mobile wireless networks is mainly dependent to the excess life of individual links. Link excess life (LEL), or link availability in some contexts, is the probability that a wireless link between two mobile nodes exists at time $t+t_0$, given that a link exists at time t_0. On the other hand, link duration is defined as the lifespan of a node-to-node link from the time a receiver enters the communication region of the transmitter to the time the receiver exits the communication region.

LEL is also an important parameter in topology control [5] in mobile networks. Some topology control protocols are highly sensitive to the link breakages and some can tolerate it [3]. However, the more knowledge we have about the LEL, the more precise the evaluation and design of a topology control protocol can be done.

LEL is also an important parameter in simulation of mobile networks. Probability density function (PDF) or cumulative distribution function (CDF) of LEL are crucial tools for generation of path availability random variable in order to simulate availability and performance of wireless networks [6].

In fact, the knowledge of LEL can serve as groundwork for further analysis of network performance, as well as a guide to ad hoc and sensor network protocol design. Although a great work is done to investigate the link duration and LEL in different scenarios [7-10], yet several aspects of this important parameter are not analyzed. In this paper, we first try to propose a model for LEL and derive a closed form expression of link excess life PDF which is a suitable tool for further analysis and simulation [6]. Then, superposition property of the LEL is introduced. One of these aspects which is not thoroughly addressed yet is the effect of stationary nodes. In this paper a systematic method to evaluate the effect of stationary nodes on the link excess life is introduced based on the superposition property. This method could be easily generalized to evaluate the LEL in the networks built of different kinds of nodes with different velocity distributions. Moreover, a general method for approximation of LEL given the PDF of relative velocity of nodes is provided. At last, the effect of buffer zone [3],[11] on advancing the LEL is investigated analytically.

2. Related work

Link duration and LEL is thoroughly investigated in the literature. In [7] link and path availability models for wireless ad hoc networks with a random walk-based

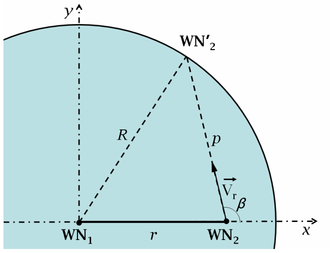

Figure 1. Relative motion of two wireless nodes

mobility are derived. In [12] a model for Brownian mobility model based on the bi-dimensional heat equation is provided. In [13] a path selection scheme based on path duration analysis results is proposed which tries to use more stable paths. This scheme is useful when the nodes use different velocity distributions. In [10] statistics of link and path duration is investigated and correlation of the reciprocal of the average path duration and throughput and overhead of reactive routing protocols is highlighted.

Numerical study of the distribution of multi-hop path durations under various mobility models in [2] shows that the distribution of path duration can be approximated by an exponential distribution when the number of hops is more than 3 or 4. In [15] this phenomenon is analytically justified based on Palm's theorem. In [17] a model for LEL is presented for multimedia streaming.

Authors in [16] introduced the residual link duration and investigated the link stability problem experimentally. Hung et.al. [6] developed a random generator for excess life in mobile telecommunication networks. In [18][20][11][19][9] different geometric models for evaluation of LEL or link duration were proposed.

3. Link excess life modeling

Our model consists of a radio propagation model, a mobility model, and a geometric model. Radio propagation model is a symmetric disk model as a common choice used in similar studies. Our mobility model is a variation of random direction model which has no boundaries and is simplified for ease of probabilistic analysis. A simplified version of this model has been used in previous studies. Here, we call it Boundless Random Direction Model (BRDM). To simulate the BRDM mobility model a toroidal region is used with side S, where $S \gg R$ (R is the transmission range) which simulates the infinite region [21]. In [18], a simplified version of this model is proposed as Constant Velocity (CV) model which allows only fixed velocity values. Briefly, we use the following assumptions:

- Nodes initially select their velocities according to a pre-specified distribution and do not change the direction or the velocity. Each node remains stationary with a probability of p_p.
- Direction of each node motion is uniformly distributed over [0,2π).
- A node's speed, its direction of motion and its location are mutually independent
- Nodes are uniformly distributed among the region with density ρ at the initial step.

3.1. The general method

To evaluate the LEL a geometric model is proposed. Consider two wireless nodes, WN$_1$ and WN$_2$ with velocity vectors $\vec{V_1}$ and $\vec{V_2}$ respectively. The relative velocity vector of WN$_2$ with respect to WN$_1$ is defined as $\vec{V_r} = \vec{V_1} - \vec{V_2}$ with magnitude $v_r \triangleq |\vec{V_r}|$. $f_{v_r}(v_r)$ is defined as the PDF of v_r with support in $[v_{r,\min}, v_{r.\max}]$. Without loss of generality, suppose WN$_1$ is considered as the origin of the orthogonal Cartesian coordinate system. In this situation, WN$_1$ can be observed stationary and WN$_2$ is moving with $\vec{V_r}$ velocity vector. Since our model is centro-symmetric, coordinate axes can rotate freely. Hence, we assume that node WN$_2$ is placed on positive X-axis (Figure 1). β is defined as the angel between the X-axis and $\vec{V_r}$. At time t_0 node WN$_1$ visits node WN$_2$ in distance r. Now, we try to evaluate the time interval in which WN$_2$ remains in the transmission region of WN$_1$ which is the excess life of the active link between WN$_1$ and WN$_2$ and is obtained by

$$\tau = P / v_r \tag{1}$$

where P is the distance which WN$_2$ must travel to reach the boundary of the transmission range of WN$_1$. First, we evaluate the CDF of P as $F_{P|r}(p_0 \mid r) = pr(P < p_0 \mid r) = pr(\beta < \beta_0 \mid r)$, where β_0 is the corresponding angel to p_0 and could be obtained by triangular relation. Hence,

$$F_{P|r}(p_0 \mid r) = F_{\beta|r}(Arc\cos(\frac{R^2 - p_0^2 - r^2}{2 p_0 r}) \mid r) \tag{2}$$

Since our mobility model is centro-symmetric, β has uniform distribution in [0,π) (this is validated by simulation) and,

$$F_{P|r}(p_0|r) = \begin{cases} 0 & p_0 \le R - r \\ \frac{1}{\pi} Arc\cos(\frac{R^2 - p_0^2 - r^2}{2 p_0 r}) & R - r \le p_0 \le R + r \\ 1 & p_0 \ge R + r \end{cases} \tag{3}$$

Recalling (1) and considering the independency of v_r and P, the CDF of τ may be obtained by

$$\begin{aligned} F_{\tau|r}(\tau_0 \mid r) &= pr(\frac{P}{v_r} < \tau_0 \mid r) \\ &= \int_{v_r} F_{P|r}(\tau_0 v_r \mid r) f_{v_r}(v_r) dv_r \end{aligned} \tag{4}$$

Differentiating on τ, we extract the PDF from CDF as:

$$f_{\tau|r}(\tau_0 \mid r) = \int_{v_r} v_r f_{P|r}(\tau_0 v_r \mid r) f_{v_r}(v_r) dv_r \tag{5}$$

In our model WN$_2$ is uniformly distributed in the transmission range of WN$_1$. Hence, the PDF of random variable r is $f_r(r) = 2r.R^{-2}$ and the PDF of excess life is obtained as:

$$f_\tau(\tau_0) = \int_{r=0}^{R} f_r(r) f_{\tau|r}(\tau_0 \mid r)\, dr \tag{6}$$

Rearranging the integrations gives:

$$\begin{gathered} f_\tau(\tau) = \int_{V_r} f_{v_r}(v_r) \int_r g(r,\tau,v_r)\, dr\, dv_r \\ g(r,\tau,v_r) \triangleq f_r(r)\, v_r f_{P|r}(\tau v_r \mid r) \end{gathered} \tag{7}$$

For a given value of τ, integration region is defined as $\Re(\tau)$ in which f_r, f_{v_r}, and $f_{P|r}$ in (7) is bounded (Figure 2) : $\Re(\tau)= \{(r,v_r) | 0 \le r \le R,\ v_{r,\min} \le v_r \le v_{r,\max}, R-r \le \tau v_r \le R+r\}$. When $v_{r,\min} = 0$, $f_\tau(\tau)$ is obtained from

$$f_\tau(\tau)= \begin{cases} h(0,v_{r,\max}, R-\tau v_r) & \tau \le T \\ h(0,R/\tau, R-\tau v_r) + h(R/\tau, v_{r,\max}, \tau v_r - R) & T < \tau < 2T \\ h(0,R/\tau, R-\tau v_r) + h(R/\tau, 2R/\tau, \tau v_r - R) & \tau \ge 2T \end{cases} \tag{8}$$

$$h(v_{\min}, v_{\max}, r_{\min}) \triangleq \int_{V_r = v_{\min}}^{v_{\max}} f_{v_r}(v_r) \int_{r=r_{\min}}^{R} g(r,\tau,v_r)\, dr\, dv_r$$

$$T \triangleq \frac{R}{v_{r,\max}}$$

From (3)-(7), the *scaling property* of $f_\tau(\tau)$ can be achieved as follows: If R is scaled by factor ω and random variable v_r is scaled by factor ψ, τ will be scaled by factor ω/ψ. The PDF of link duration can be derived from PDF of LEL as [6]

$$f_L(\tau) = \frac{df_\tau(\tau)}{d\tau} E[L] \tag{9}$$

where, L is the link duration with F_L as CDF. $E[L]$ may be easily derived by applying Little theorem [9][8].

4. Constant velocity model

If all the nodes move with a constant velocity ($|\vec{V}| = 1$), like the model proposed in [9], the magnitude of relative velocity vector of the two nodes has a distribution of form:

$$f_{v_r}(v_r) = \begin{cases} \dfrac{2}{\pi.\sqrt{4 - v_r^{\,2}}} & 0 \le v_r \le 2 \\ 0 & otherwise \end{cases} \tag{10}$$

Using (8) the following PDF is obtained for LEL (Figure 3):

$$f_\tau(\tau) = \frac{1}{\pi^2 R^2 \tau}[4\tau R + 2(R-\tau)(R+\tau)\ln(\frac{R+\tau}{|R-\tau|})] \tag{11}$$

Regarding the scaling property of $f_\tau(\tau)$, if nodes move with velocity of $|\vec{V}| = v_{cte}$, the PDF of LEL is obtained by $f_\tau(\tau v_{cte}) v_{cte}$ and link duration is derived from (9) as

$$f_L(L) = \frac{df_\tau(\tau)}{d\tau} . \frac{\pi^2 . R}{8} \tag{12}$$

The average LEL is obtained by $E[\tau] = \int_{\tau=0}^{+\infty} \tau f_\tau(\tau) d\tau$ which is *infinite* for the constant velocity model. This interesting result shows that the average LEL is not a stable mobility metric and is not proportional to the average link duration. On the other hand, the average link duration is proposed as a performance or connectivity metric [9], [14]. However, this result shows it is not a suitable metric at least to reflect the availability of single hop paths.

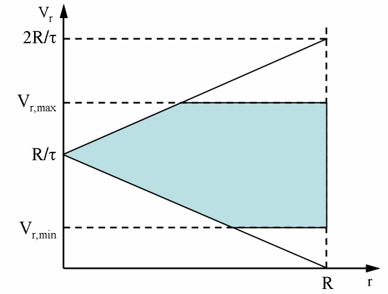

Figure 2. Integration region

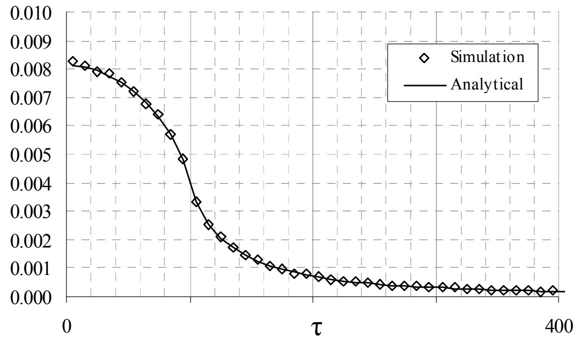

Figure 3 Analytical and experimental link excess life PDF for constant velocity model. (v_{cte}=1m/s, R=100m)

5. Superposition property of LEL

Suppose the PDF of the relative velocity could be expressed as a weighted summation of M PDFs as:

$$f_{v_r}(v_r) = \sum_{k=1}^{M} c_k \hat{f}_{v_r,k}(v_r) \tag{13}$$

It is obvious that summation of c_k values must be 1. Substituting the above expression in (7) gives:

$$f_{\tau}(\tau) = \int_{V_r} \sum_{k=1}^{M} c_k \widehat{f}_{v_r,k}(v_r) \int_{r} g(r,\tau,v_r)\,dr\,dv_r$$
$$= \sum_{k=1}^{M} c_k f_{\tau,k}(\tau) \tag{14}$$

where $f_{\tau,k}(\tau)$ is the PDF of LEL corresponding to the $\widehat{f}_{v_r,k}(v_r)$ relative velocity PDF. This property is held for the CDF of LEL, too.

5.1. Approximation of excess life

In fact, numerically evaluating the PDF of excess life by (8) requires a heavy computation. In this section we propose a method to approximate the excess life PDF based on its superposition property.

The main idea is converting $f_{v_r}(v_r)$ into a set of $\widehat{f}_{v_r,k}(v_r)$ functions which have pre-calculated $f_{\tau,k}(\tau)$ functions. Here, we calculate $f_{\tau,k}(\tau)$ for the Dirac delta function and decompose $f_{v_r}(v_r)$ into a set of delta functions. In other words, we try to approximate the continuous random variable v_r by a discrete random variable $\tilde{v}_r$. Thus,

$$f_{\tilde{v}_r}(v_r) = \sum_{k=1}^{M} c_k \delta(v_r - x_k), \quad c_k = \int_{x_k-\Delta/2}^{x_k+\Delta/2} f_{v_r}(v_r)\,dv_r \tag{15}$$

where, $\Delta \triangleq (v_{r,\max} - v_{r,\min})/M$, $x_k \triangleq v_{r,\min} + \Delta/2 + (k-1)\Delta$ and M is a positive integer. The $f_{\tau,k}(\tau)$ corresponding to $\widehat{f}_{v_r,k}(v_r) = \delta(v_r - x_k)$ is obtained by

$$f_{\tau,k}(\tau) = \begin{cases} \dfrac{x_k\sqrt{4R^2 - \tau^2 x_k^{\,2}}}{\pi R^2} & \tau < \frac{2R}{x_k} \\ 0 & \mathit{otherwise} \end{cases} \tag{16}$$

Figure 4 depicts a sample of the approximation method with $M = 25$.

5.2. Effect of stationary nodes

Consider each node is stationary (or pausing for a long time) with probability p_p. A randomly selected link has three situations: both ends are stationary nodes, only one end is a stationary node, or both ends are moving. Hence, the relative velocity PDF is:

$$f_{v_r,p}(v_r) = p_p^{\,2}\,\delta(v_r) + 2p_p(1-p_p)f_v(v) + (1-p_p)^2 f_{v_r}(v_r) \tag{17}$$

where $f_{v_r,p}(v_r)$ and $f_{v_r}(v_r)$ are respectively the PDFs of relative velocity with and without considering stationary nodes and $f_v(v)$ is the PDF of individual node velocity values. The PDF of global LEL would be achieved from (17) by applying superposition property. Note that when both ends of a link are stationary (first term of (17)), the LEL will be infinite for this category of links and its PDF could be expressed as a weighted Dirac delta function at infinity. In constant velocity model, $f_v(v) = \delta(v - v_{cte})$, hence the PDF of LEL for the category of links with just one stationary end is achieved from (16). For uniform velocity distribution in (0,2], we have $f_{\tau}(\tau) = 4(3\pi\tau^2)^{-1} \times [R - (R^2 - \tau^2)^{3/2} u(R - \tau)]$ (u is unit step function) for this category of links. Figure 5 depicts a sample of this method. As described before a Dirac delta function at infinity exists because of the presence of stationary links which is not depicted in the figure.

This approach could be generalized for networks with several categories of nodes with different velocity PDFs such as vehicles, bicycles, and pedestrians. The effect of each category on the global LEL could be exploited.

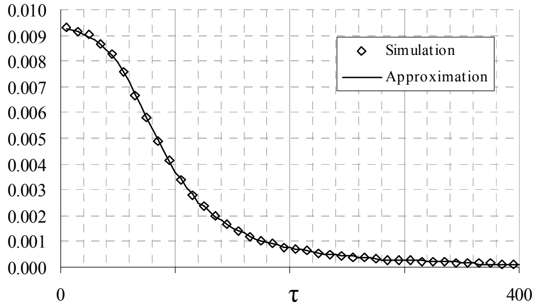

Figure 4. Simulation and approximation of PDF of LEL in BRDM model with R=100m and velocity selected uniformly from (0,2] m/s.

6. Effect of buffer zone

The buffer zone is introduced in [3] to improve the topology control protocols in mobile networks. The idea is sending the "Hello" message with radius R_1 and constructing the topology, and then setting the transmission range to R>R_1 which leads to a more durable connected topology. The PDF of LEL is an important parameter for the constructed topology which indirectly tells how long the constructed topology is expected to be connected after execution of topology control protocol.

The method mentioned in Section with a slight modification is applicable here. $f_r(r)$ is changed to $f_r(r) = 2r.R_1^{-2}$ for $0 < r < R_1$ and integration region is constrained to $0 < r < R_1$. Resulted PDF for constant velocity model for $R = 100$ and various values of R_1 is depicted in Figure 6. As shown in the figure, for a period of $(R - R_1)/v_{r,\max}$ no link is broken since the topology surely remains connected in this interval. But after this period the link loss probability is considerably increased to more than

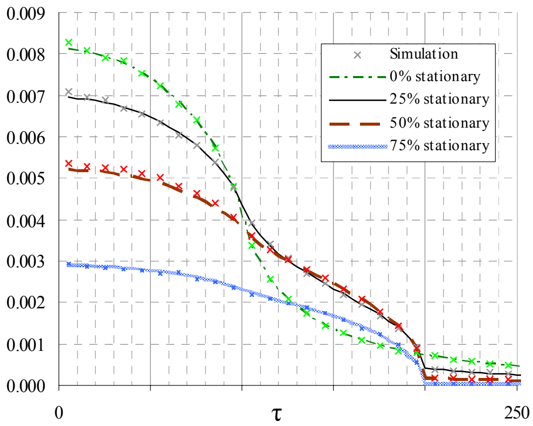

Figure 5. Effect of the number of stationary nodes on the link excess life PDF in constant velocity model with v=1m/s and R=100m. (Consider a weighted delta function at infinity as described in the context)

that of the case without buffer zone. Indeed, the two curves for values of $\tau > R$ are roughly the same. Therefore, it can be concluded that the buffer zone has only short term effects on the connectivity.

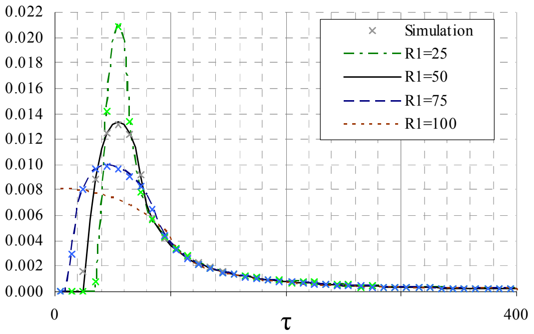

Figure 6. Effect of different buffer zones on the link excess life PDF in constant velocity model with v=1m/s and R=100m.

7. Conclusions

In this paper, the link excess life PDF as an important parameter in performance and connectivity evaluation of wireless mobile networks is investigated. A mobility model (BRDM) was introduced for ease of probabilistic evaluations. A geometric model was also presented to extract expressions for link excess life PDF. The link excess life PDF for constant velocity model is investigated leading to some closed form expressions. Analysis of this expression made an interesting result: the average LEL is infinite, in spite of average link duration which is a finite value. Superposition of link duration PDF from relative velocity distribution is introduced which had several outcomes like approximation of the link excess life PDF and investigation of the effect of stationary nodes on the global link excess life PDF. The effect of buffer zone in increasing the durability of network connectivity after execution of topology control protocols is also investigated from viewpoint of link durability. Outcomes show that the buffer zone has a perfect effect on short term connectivity. To validate the obtained results extensive simulation is performed and accurate results are obtained.

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