Analysis of Time-Based Random Waypoint Mobility Model for Wireless Mobile Networks

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Abstract

In this paper Time-Based Random Waypoint (TBRWP) model is analyzed in detail. Random waypoint model has been studied in mobility literature extensively. Traditional RWP model is based on the independency of the velocity and trajectory; TBRWP, instead, focuses on the independency of motion time and trajectory. Basic model is introduced and general method to evaluate the asymptotical probability density function (PDF) is presented. The model for the one-dimensional and two-dimensional cases with uniform destination distribution in unit square area is solved and exact closed form formulas is obtained. Moreover, normal distribution of nodes in one- and two- dimensional models is investigated and practical approximations are derived for them. To verify the obtained results an extensive simulation is performed and tight results are obtained

1. Introduction

Wireless networks are growing rapidly and will have much effect in our future life. Thus, performance analysis in the presence of different factors is an important issue. One of these factors that influences the design and analysis of those networks is mobility [1][2][3]. Many different models have been proposed by researchers such as random waypoint (RWP) model [4], random directional model [5], Brownian motion [6], and map based mobility [7][8][9].Based on these models many different results have been extracted for various network architectures [10][11].

RWP is one of the most common models for mobile networks. In this model, each node chooses uniformly at random a destination point within the deployment region R, and moves toward it along a straight line. Node velocity is chosen randomly with uniform distribution in the interval $[v_{min}, v_{max}]$. When the node arrives at destination, it remains stationary for a predefined pause time, and then starts moving again according to the same pattern.

Time-Based Random Waypoint (TBRWP) mobility model [12] is a variation of RWP that solves several problems of RWP. For example, decaying average node velocity problem [13] and dependency of average node velocity (and corresponding evaluated performance measures) to the exact distribution of motion steps [14]. Moreover, TBRWP is closer to real scenarios in which objects selects their velocity according to the travel length [12]. In this paper, TBRWP is analyzed thoroughly for several conditions. In TBRWP, the key idea is choosing the motion time independently of the trajectory length. In this scenario, each mobile node selects a motion time according to an arbitrary distribution over [Tmin, Tmax], for finite values of Tmin and Tmax, and adjusts the motion velocity according to it. In case of fixed time motion time, nodes select their velocity proportional to the trajectory length.

In the next section we will introduce our model precisely. In Section 3 we analyze one-dimensional and two-dimensional regions with uniform destination distribution. Section 4 discusses the case for one and two dimensional normal node distribution. Section 5 describes the simulation method used to verify the results, and finally we conclude our paper.

2. Model description

In this model there are N points distributed in the region, which move and pause independently. After every motion step, each node goes to pause state for tp time units then chooses a new random destination point and repeats this pattern. Motion time, Tm, is a random variable bounded in $[T_{min}, T_{max}]$, with a well-defined expected value E[Tm]. It is important to assume that the start time of each node motion is independent of the other ones. In other words, considering the motion times as a random process, sample paths must be mutually independent. In case of fixed Tm, an iid initial pause time with uniform distribution in $[0, T_m]$ may be considered to achieve motion start time independence. Since each node moves and pauses independently of the other ones, it is sufficient to investigate motion of a single node to achieve asymptotical distributions. Notations summary is presented in Table 1.

Table1 Notations summary

Notation	Meaning
N_d	Number of dimensions
$\vec{s}_i$	Source of the i[th] motion step, $\vec{s}_i = \vec{d}_{i-1}$
$\vec{d}_i$	Destination waypoint of the i[th] motion step, $\vec{d}_0$ is the initial node position
$\vec{l}\,(t)$	Location of the node at time t
$T_{m,i}$	Motion time of the i[th] motion step
$T_{p,i}$	Pause time of the i[th] motion step
t_i	Start time of the i[th] motion step
τ	Time offset in the current motion step
$\hat{\tau}$	Normalized τ
$\vec{l}_i\,(\tau)$	Location in the i[th] motion step at time offset τ
$\vec{l}_{n,i}\,(\hat{\tau})$	Location in the i[th] motion step at normalized time offset $\hat{\tau}$
$\vec{l}_{n,i,j}\,(\hat{\tau})$	The j[th] dimension of $\vec{l}_{n,i}\,(\hat{\tau})$
$f_{T_p}(t_p)$	PDF of pause time
$f_{\vec{l}_{n,i}\vert\hat{\tau}}(\vec{l}\mid\hat{\tau})$	PDF of node location in the i[th] motion step at a given normalized time offset
$f_{\vec{l}\vert\hat{\tau}}(\vec{l}\mid\hat{\tau})$	Asymptotic PDF of node location at a given normalized time offset
$f_{\vec{l}}(\vec{l})$	Asymptotic location PDF
$f_{\hat{\tau}}(\hat{\tau})$	Asymptotic PDF of normalized time offset

The movement step of each node is indexed by i. Here, we use $\vec{d}_i$ to present a random vector which corresponds to the position of the destination point of the i[th] motion step in the N[d]-dimensional space. It is obvious that $\vec{s}_i = \vec{d}_{i-1}$ for $i>0$. $\vec{d}_0$ is the initial node position and has the same distribution as the destination node in each step. $T_{m,i}$ and $T_{p,i}$ are motion time and pause time of i[th] step, respectively. t_i stands for the start time of the i[th] step. With these considerations, TBRWP model can be considered as a stochastic process $\{\vec{l}\,(t)\}$ where t is the time and $\vec{l}\,(t)$ is a random vector which represents the location of the node at time t. Each value of t belongs to a time range $[t_i, t_{i+1})$ of the i[th] step for a value of i. This value of *i* will be referred to as I(t). We will refer to the time offset $t-t_i$ as τ. Like [15], we assume that the pause time after each movement period is chosen from an arbitrary PDF $f_{T_p}(t_p)$ in the interval $[T_{p,\min}, T_{p,\max}]$ with $T_{p,\min} \geq 0$ and a well-defined expected value $E[T_p]$.

According to the above considerations we can write the sample path function in the i[th] step as:

$$\vec{l}_i\,(\tau) = \vec{s}_i + (\vec{d}_i - \vec{s}_i)\ \tau \big/ T_{m,i} \qquad (1)$$

Which is the equation of a linear motion from a source point $\vec{s}_i$ to a destination $\vec{d}_i$. Note that with normalizing the time offset $\hat{\tau} = \tau / T_{m,i}$, ($\hat{\tau}$ is normalized τ) the normalized equation is obtained:

$$\vec{l}_{n,i}\,(\hat{\tau}) = \vec{l}_i\,(\hat{\tau}.T_{m,i}) = \vec{s}_i + (\vec{d}_i - \vec{s}_i)\,\hat{\tau} \qquad (2)$$

In case of square fields with uniform distribution of nodes, each element of of $\vec{s}_i$ and $\vec{d}_i$ is an independent uniform random variable. Moreover, the mobility model of the node image on each axis is the same. That is, the image on each dimension axis chooses a uniformly distributed source and destination on this axis and moves in T_m from source to destination. This is a key feature that helps to simplify the evaluation of probability distribution function of $\vec{l}_i\,(\tau)$ in high dimensional spaces. Indeed, we have:

$$f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l} \mid \hat{\tau}) = \prod_{j=1}^{Nd} f_{l_{n,i,j}|\hat{\tau}}(l_j \mid \hat{\tau}) \qquad (3)$$

Where $l_{n,i,j}\,(\hat{\tau})$ stands for the j[th] dimension of $\vec{l}_{n,i}\,(\hat{\tau})$ and $f_{l_{n,i,j}|\hat{\tau}}(l_j \mid \hat{\tau})$ is the corresponding PDF that has the same equation for each dimension.

2.1 General analytical method

Here, we present a general method to evaluate the asymptotical probability density function for a given region and waypoint distribution. The general method is based on (1). Indeed, equation (1) is a linear function of two random vectors $\vec{s}_i$ and $\vec{d}_i$. Hence,

$$\vec{l}_{n,i}\,(\hat{\tau}) = (1-\hat{\tau})\vec{s}_i + \hat{\tau}\vec{d}_i \qquad (4)$$

Using convolution or Jacobean method, $f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l} \mid \hat{\tau})$ can be obtained from waypoint distribution. Since all the motion steps are equivalent, $f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l} \mid \hat{\tau})$ is independent of *i*, and,

$$f_{\vec{l}|\hat{\tau}}(\vec{l} \mid \hat{\tau}) = f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l} \mid \hat{\tau}) \qquad (5)$$

The asymptotic PDF $f_{\vec{l}}(\vec{l})$ can be obtained by

$$f_{\vec{l}}(\vec{l}) = \int_0^1 f_{\vec{l}|\hat{\tau}}(\vec{l} \mid \hat{\tau}).f_{\hat{\tau}}(\hat{\tau})\,d\hat{\tau} \qquad (6)$$

Considering the ergodic property of the random process, $F_{\hat\tau}(t) = \mathrm{P}(\hat\tau < t)$ can be interpreted as the limit of the ratio of total time in which $\hat\tau$ is less than t to the total elapsed time as

$$F_{\hat\tau}(t) = \lim_{S\to\infty} \frac{1}{S.E[T_m]} \sum_{i=1}^{S} \frac{t}{T_{m,i}} = t \qquad (7)$$

Where S is the number of steps and tends to infinity. Therefore, $\hat\tau$ has uniform asymptotic distribution. Recalling (6), $f_{\vec l}(\vec l)$ can be is obtained as:

$$f_{\vec l}(\vec l) = \int_0^1 f_{\vec l|\hat\tau}(\vec l \mid \hat\tau)\, d\hat\tau \qquad (8)$$

This result shows that the asymptotical PDF of node location is independent of motion time distribution. In other words, motion time distribution is arbitrary but must have a well-known expected value. To consider pause time, the method mentioned in [15] could be used which results in:

$$f_{\vec l_{mp}}(\vec l) = f_{\vec l}(\vec l)\frac{\bar t_m}{\bar t_m + \bar t_p} + f_{\vec D}(\vec l)\frac{\bar t_p}{\bar t_m + \bar t_p} \qquad (9)$$

Where $\bar t_m$, $\bar t_p$, and $f_{\vec D}(\vec l)$ stand for average motion time, average pause time, and distribution of destination(source) points respectively. Ergodicity of the random process could be achieved through the method mentioned in [15].

2.3 Simulation method

To verify the analytical results an extensive simulation has been performed by XMulator [16] (which is designed and extended to support mobility models by the authors). The simulation method is based on dividing the region into small square parts (20 slices in each axis), moving the node according to the model, observing the node position in time slices, and counting the number of times that the node is observed in each square part. Simulation results for different cases are depicted in the next sections.

3. Uniform waypoint distribution model

According to the previously mentioned general method, the evaluation of one and two dimensional regions with uniform waypoint distribution is presented here.

3.1 One-Dimensional region

In this model, waypoints are chosen uniformly from [-1/2, 1/2] interval and $\vec l = [x]$. According to (4), the PDF of $\vec l_{n,i}(\tau)$, i.e. location of the node in step i at normalized time offset τ, could be achieved through convolution of two uniform distributions:

$$f_{\vec l_{n,i}|\hat\tau}(\vec l \mid \hat\tau) = f_{\vec l_{n,i}|\hat\tau}(x \mid \hat\tau) = \frac{1}{2\tau(\tau-1)}[(2\tau-1)U(x+\tau-\tfrac{1}{2}) + 2(x-\tau)U(x-\tau+\tfrac{1}{2}) - U(\tau-x-\tfrac{1}{2}) - 2xU(\tfrac{1}{2}-x-\tau)] \qquad (10)$$

where, $U(x)$ stands for the unit step function and τ is normalized time offset in [0, 1]. Figure 1 depicts $f_{\vec l_{n,i}|\hat\tau}(\vec l \mid \tau_n)$ for several values of normalized time offset. For values of τ near zero, the position of $\vec l$ has uniform distribution, which is interpreted by the initial distribution of the source node. When τ increases to 1/2, the curve tends to the triangular distribution. For the values of τ more than 1/2, PDF tends to uniform distribution again.

Applying (8) to the above result gives the PDF for a specified value of x as:

$$f_x(x) = 2\ln 2 - (1+2x)\ln(1+2x) - (1-2x)\ln(1-2x) \quad (11)$$

Figure 2 shows the $f_X(x)$ curve, corresponding simulation result, and PDF of traditional RWP model. Figure 3 illustrates the effect of applying a pause time equal to the motion time in average.

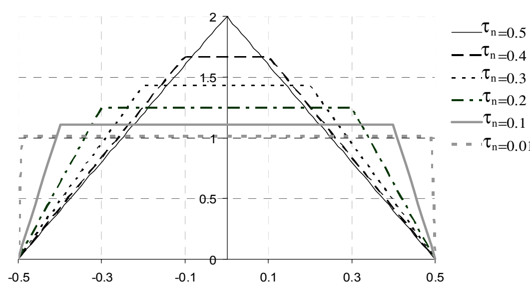

Figure 2. Temporal locations PDF of node for different values of (normalized time offset) in one dimensional region with uniform node distribution.

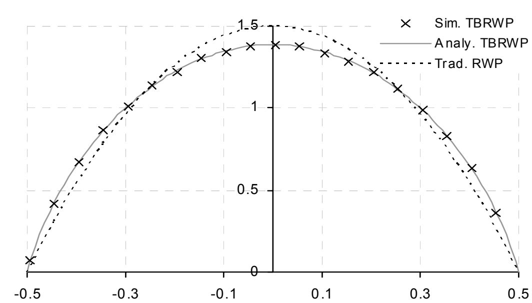

Figure 3. Asymptotical PDF of TBRWP model, corresponding simulation result, and PDF of traditional RWP model.

3.2 Two-Dimensional region.

Here, waypoints are chosen uniformly from $\{(x,y) \mid -1/2 \le x \le 1/2,\ -1/2 \le y \le 1/2\}$. For simplicity, $\hat\tau$ is shifted to [-1/2, 1/2].

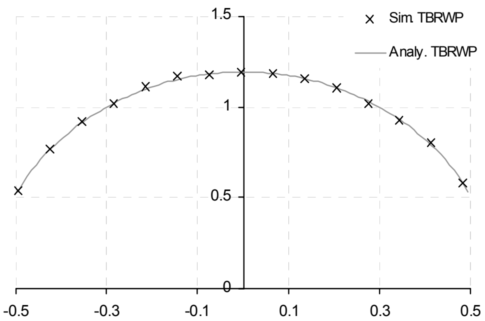

Figure 1. One dimensional uniform model with pause time equal to motion time in average.

According to the (3), (4), and (10), the PDF of random variable $\vec{l}_{n,i}(\hat{\tau})$ is obtained by

$$f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l}\,|\,\hat{\tau}) = \prod_{j=1}^{Nd} f_{l_{n,i,j}|\hat{\tau}}(l_j\,|\,\hat{\tau}) = f_{l_{n,x}|\hat{\tau}}(x\,|\,\hat{\tau}) f_{l_{n,y}|\hat{\tau}}(y\,|\,\hat{\tau}) \tag{12}$$

Since this function is symmetric, evaluation of the function for $\{(x,y)\,|\; 0 \le x \le 1/2,\, 0 \le y \le 1/2,\, y \le x\}$ is sufficient. Using (8) the PDF for each pair of (x,y) is obtained as

$$f_{x,y}(x,y) = 2-4x+(1-2x)[2y\ln(\frac{1-2y}{1+2y})+\ln(\frac{1+2x}{1-2x})] \quad ,0 \le y \le x \le 1/2 \tag{13}$$

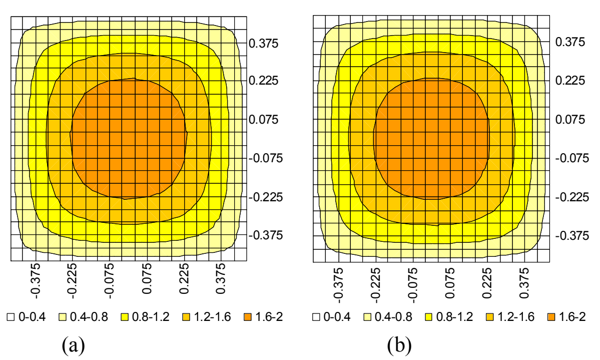

Figure 4. PDF of Two dimensional TBRWP model with uniform waypoint distribution. (a) Analytical results (b) simulation results.

4. Normal waypoint distribution model

In this section we analyze the asymptotical PDF for an infinite region with normal distribution of waypoints around the center. This model could be used in applications with node clustering around some points [17][18]. To achieve closed form formulas some approximations are used and the region is limited to $[-3\sigma, +3\sigma]$. (Actually, integration of normal distribution PDF in this range yields $erf\,(3\sqrt{2}/2) = 0.997$).

4.1 One-Dimensional region

In this model, waypoints are distributed with $N(0,\sigma)$ distribution and $\vec{l}$ is a one dimensional vector. According to (4), $\vec{l}_{n,i}(\hat{\tau})$ is summation of two scaled normal random variables which has normal distribution, too:

$$f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l}\,|\,\hat{\tau}) = f_{\vec{l}_{n,i}|\hat{\tau}}(x\,|\,\hat{\tau}) \sim N\left(0, \sigma\sqrt{(\frac{1}{2}+\hat{\tau})^2+(\frac{1}{2}-\hat{\tau})^2}\right) \tag{14}$$

Where, $\hat{\tau}$ is normalized time offset shifted to [-1/2, 1/2]. To integrate the $f_{\vec{l}_{n,i}|\hat{\tau}}(x\,|\,\hat{\tau})$ over $\hat{\tau}$, an approximation method is used. Considering $\sigma = 1$, by substituting $1/\sqrt{4\tau^2+1}$ with θ and evaluation of first 3 sentences of Taylor series around $\theta = 1$, the above expression could be approximated. Integration of the approximated expression in range $\hat{\tau} \in [-1/2, 1/2]$ gives:

$$f_X(x,\sigma=1) \approx \hat{f}_X(x) = \frac{1}{4\sqrt{\pi}} e^{-x^2}[-4C_1+(-16C_1-4-3\pi)x^2+(16C_1+8+2\pi)x^4] \tag{15}$$

Where, $C_1 = \ln(\sqrt{2}-1)$ and $\hat{f}_X(x)$ is the approximated PDF. $f_X(x)$ could be approximated by $\hat{f}_x(x/\sigma)/\sigma$, for different values of σ. Figure 5 shows the simulation result and approximated PDF. Numerical integration of absolute error in the range gives:

$$\int_{-3\sigma}^{3\sigma} |f_x(x) - \frac{\hat{f}_x(x/\sigma)}{\sigma}|\,dx \approx 0.0058 \tag{16}$$

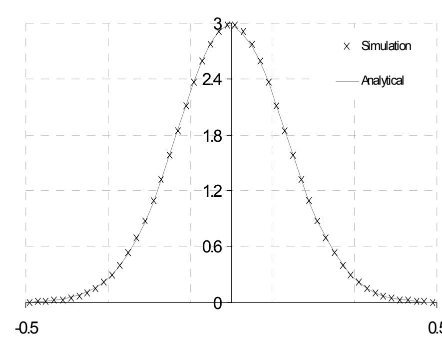

Figure 5. Asymptotical PDF of one dimensional TBRWP model for normal waypoint distribution.

4.2 Two-Dimensional region

Here, waypoints are chosen from $\{(x,y) \mid x \sim N(0,1),\ y \sim N(0,1)\}$, $\vec{l} = (l_x, l_y)$, and $\hat{\tau}$ is shifted to [-1/2, 1/2]. The PDF of $\vec{l}_{n,i}(\hat{\tau})$ is as

$$f_{\vec{l}_{n,i}|\hat{\tau}}(\vec{l}) = f_{l_{n,x}|\hat{\tau}}(x \mid \hat{\tau}) f_{l_{n,y}|\hat{\tau}}(y \mid \hat{\tau}) = \frac{e^{-\frac{x^2+y^2}{\sigma^2(4\hat{\tau}^2+1)}}}{\sigma^2\pi(4\hat{\tau}^2+1)} \quad (17)$$

It is more convenient to represent it in polar coordinates:

$$f_{\vec{l}_{n,i}|\hat{\tau}}(r \mid \hat{\tau}) = 2\pi r \frac{1}{\sigma^2\pi(4\hat{\tau}^2+1)} e^{-\frac{r^2}{\sigma^2(4\hat{\tau}^2+1)}} \quad (18)$$

Similar to the one-dimensional case, approximation is used by substituting $1/(4\hat{\tau}^2+1)$ with θ and evaluating of first 3 sentences of Taylor series around $\theta = 1$. Integration of the approximated expression in range $\hat{\tau} \in [-1/2, 1/2]$ gives

$$f_r(r) \approx \hat{f}_r(r) = \frac{r}{8\sigma^2} e^{-(\frac{r}{\sigma})^2} [4\pi + (-4+2\pi)(\frac{r}{\sigma})^2 + (10-3\pi)(\frac{r}{\sigma})^4] \quad (19)$$

where $\hat{f}_r(r)$ is the approximated PDF. Considering the symmetric property of the PDF, $f_{x,y}(x,y)$ could be achieved by:

$$\hat{f}_{x,y}(x,y) = \frac{1}{2\pi r}\hat{f}_r(r)\Big|_{r=x^2+y^2} \quad (20)$$

Numerical integration of absolute error in the range $[-3\sigma, 3\sigma]$ gives:

$$\int_{-3\sigma}^{3\sigma}\int_{-3\sigma}^{3\sigma} |f_{x,y}(x,y) - \hat{f}_{x,y}(x,y)|\, dx\, dy \approx 0.017 \quad (21)$$

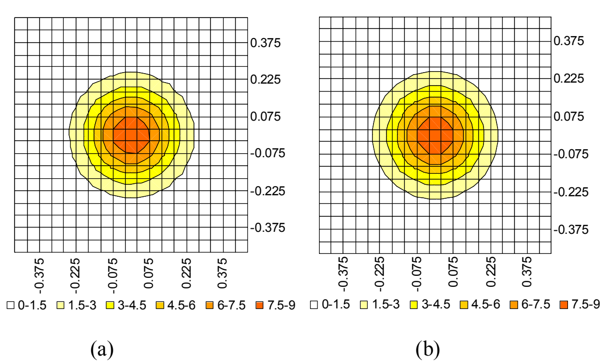

Figure 6. PDF of Two dimensional TBRWP model with uniform waypoint distribution (a) Simulation results (b) analytical results.

And for square error we have:

$$\sigma^2 \int_{-3\sigma}^{3\sigma}\int_{-3\sigma}^{3\sigma} \left[f_{x,y}(x,y) - \hat{f}_{x,y}(x,y)\right]^2 dx dy \approx 0.000013 \quad (22)$$

5. Conclusions

Time-Based RWP is a variation of traditional RWP that focuses on selection of motion time value independently of the trajectory length while the traditional RWP focuses on selection of motion velocity independently of the trajectory length. TBRWP can solve several problems of RWP. In this paper, a detailed analysis of TBRWP is presented. To obtain the asymptotic PDF of nodes, which is one of the most important properties of any mobility model, an analytical model is proposed. This model for one-dimensional and two-dimensional fields is investigated for uniform and normal waypoint distributions. In normal case, some approximation methods are used to achieve two closed form equations. Finally, theoretical results are validated through simulation experiments

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